

(revised 12/30/05)

RAMSAUER - TOWNSEND EFFECT

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Abstract

The scattering cross section of electrons on noble gas atoms exhibits a very small value at electron energies near 1 eV. This is the Ramsauer-Townsend effect and provides an example of a phenomenon which requires a quantum mechanical description of the interaction of particles.

Theory

The Ramsauer-Townsend effect can be observed as long as the scattering does not become inelastic by excitation of the first excited state of the atom. This condition is best fulfilled by the closed shell noble gas atoms. Physically, the Ramsauer-Townsend effect may be thought of as a diffraction of the electron around the rare-gas atom, in which the wave function inside the atom is distorted in just such a way that it fits smoothly to an undistorted wave function outside. The effect is analagous to the perfect transmission found at particular energies in one-dimensional scattering from a square well. Appendix A (from Ref.[2]) contains a one-dimensional treatment of scattering from a square well. This is the first model which you will use to analyze the data. A three-dimensional treatment using partial waves is given in Ref. [4], pp 396-402.

Apparatus

Thyratron - (RCA 2D21)

The tube contains Xenon gas. The assembly is mounted on a stand so that the filament of the tube is uppermost and so that the tube may be dipped into a liquid nitrogen dewar. (Note that the voltages being used here are NOT the voltages which are normally used in thyratron circuits).

Regulated DC Power Supply

The supply provides the voltage to accelerate the electrons. The supply provides 0 to 30 volts but is difficult to adjust for very low voltages. For this reason a control box containing a potentiometer is used to accurately set the lower voltages.

4-Volt Transformer

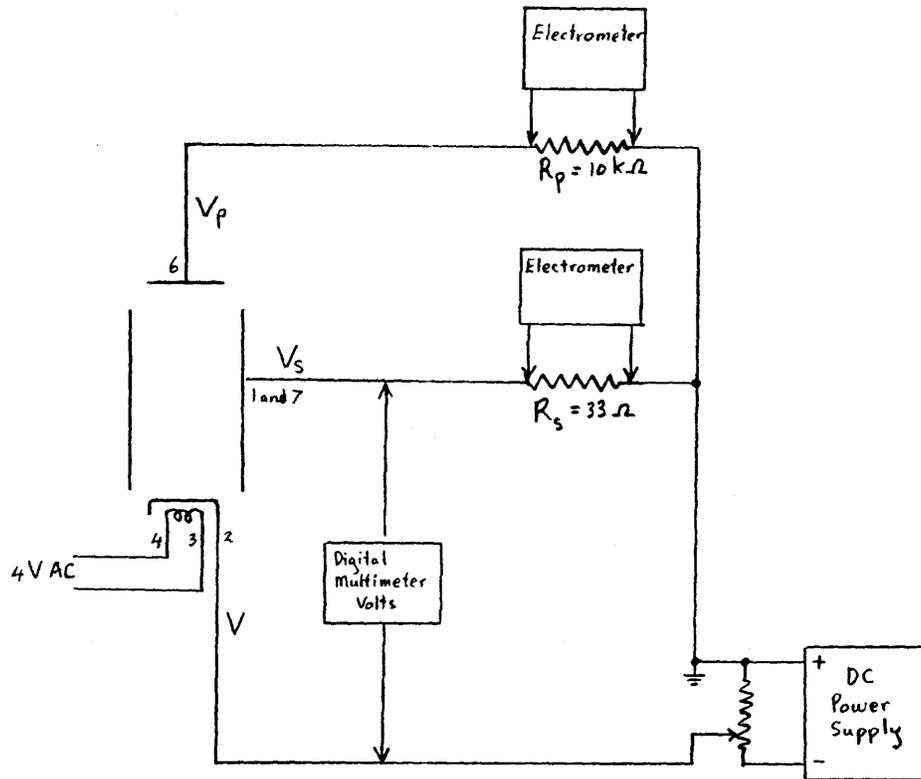
The transformer provides the power for the thyratron filament. The tube normally uses 6.3 volts AC but by running the cathode at a lower temperature the spread in electron energies is reduced. The transformer is contained in the control box.

Dewar Flask

The dewar will hold the liquid nitrogen necessary for freezing out the Xenon in the thyratron tube. The cold data is used to correct for thyratron geometry effects.

Digital Multimeters - (3 1/2 digit Data Precision 1450)

These are high impedance meters used to measure the plate voltage, V_p ; the shield voltage, V_s ; and the cathode to shield voltage, $(V - V_s)$.



Circuit

Diagram

Thratron Socket Wiring Color Code

Pin	Internal Connection	Color of Wire
1	grid #1	green*
2	cathode	black
3	heater	red
4	heater	red
5	shield (grid #2)	no connection
6	anode	yellow
7	shield (grid #2)	green*

* grid #1 and shield (grid #2) are joined externally

Demonstration of the Ramsauer-Townsend Effect in a Xenon Thyatron

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(Received 15 January 1968; revision received 15 April 1968)

The anomalously small scattering of electrons near 1 eV energy by noble gas atoms may be easily demonstrated using a 2D21 xenon thyatron. This experiment is suitable for a lecture demonstration or for an undergraduate physics laboratory. The probability of scattering and the scattering cross section may be obtained as a function of electron energy by measuring the grid and plate currents in the tube.

The scattering cross section for electrons on noble gas atoms exhibits a very small value at electron energies near 1 eV. This cross section is much smaller than that obtained from measurements involving atom-atom collisions. This is the Ramsauer-Townsend effect and provides an example of a phenomenon which requires a quantum mechanical description of the interaction of particles. If the atoms are treated classically as hard spheres, the calculated cross section is independent of the incident electron energy and we cannot account for the Ramsauer-Townsend effect. If the noble-gas atoms are considered to present an attractive potential (e.g., square well, screened Coulomb) of typical atomic dimensions, the solution of the Schrödinger equation for the electrons indicates that the cross section will have a minimum at electron energies near 1 eV. Reviews of the Ramsauer-Townsend effect are given by Mott and Massey¹ and Brode.²

¹N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, London, 1965), 3rd ed., Chap. 18.

²R. B. Brode, *Rev. Mod. Phys.* **5**, 257 (1933).

The problem of scattering of electrons by a square well is considered in many introductory quantum physics texts.³⁻⁷ The one-dimensional model predicts that the scattering will go to zero whenever half the electron wavelength in the well is a multiple of the well width. The difficulty with this model is that only one distinct minimum is observed.

A slightly better model of the xenon atom is a three dimensional square well. Then the scattering cross section will have a very small value when the phase shift δ_0 of the $l=0$ partial wave is π . Here the scattering due to the $l=0$ partial wave will vanish and the scattering due to higher l partial waves will be small if the width of the

³L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Co., New York, 1955), Chap. 5.

⁴E. Merzbacher, *Quantum Mechanics* (John Wiley & Sons, Inc., New York, 1955), Chaps. 6, 12.

⁵D. Bohm, *Quantum Theory* (Prentice-Hall Inc., Englewood Cliffs, N.J., 1951), Chaps. 11.9, 21.51.

⁶A. Messiah, *Quantum Mechanics I* (North-Holland Publ. Co., Amsterdam, 1961), Chaps. III-6.

⁷R. M. Eisberg, *Fundamentals of Modern Physics* (John Wiley & Sons, Inc., N.Y., 1961), Chap. 15.

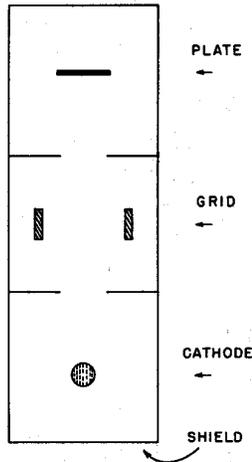


FIG. 1. Cross section of the 2D21 thyratron.

well is small.¹ When the $l=0$ phase shift becomes 2π , or higher l phase shifts become π at higher values of electron energy, the dips in the cross section will not be as prominent since contributions from other values of l will not be small. The well parameters may be adjusted to give a minimum at the observed energy. This model predicts the Ramsauer-Townsend effect in a qualitative way, but does not give quantitative agreement over a wide range of electron energies. The results of more accurate calculations with a screened coulomb potential are given by Mott and Massey.¹

I. THE EXPERIMENT

The 2D21 thyratron is very well suited for a demonstration of the Ramsauer effect. The shield (grid 2) is a boxlike structure with three sections

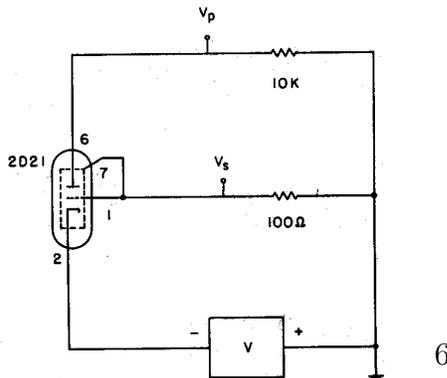


FIG. 2. Diagram of the circuit for the Ramsauer effect experiment. The filament of the 2D21 (pins 3, 4) is heated by 4 V dc.

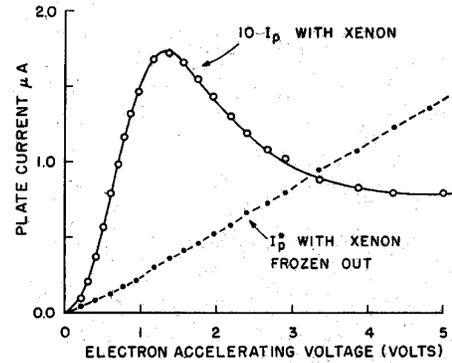


FIG. 3. The plate current I_p as a function of the voltage V , and I_p^* the plate current with the xenon frozen out with liquid nitrogen.

connected by apertures (see Fig. 1). The electron beam originates at the cathode in the first section, passes through the second section, and part of it is collected on the plate in the third section. The xenon pressure in the tube is approximately 0.05 Torr. A diagram of the circuit is shown in Fig. 2. The shield current is proportional to the intensity of the electron beam at the first aperture. After the first aperture the beam passes through an equipotential region where the scattering takes place. In this region the beam intensity is $J = J_0 e^{-x/\lambda}$, where λ is the mean free path. If the plate is a distance l from the first aperture, the intensity at the plate is $J_p = J_0 e^{-l/\lambda}$ or $J_p = J_0(1 - P_s)$, where P_s is the probability of scattering. The plate current is $I_p = I_s f(V)(1 - P_s)$, where I_s is

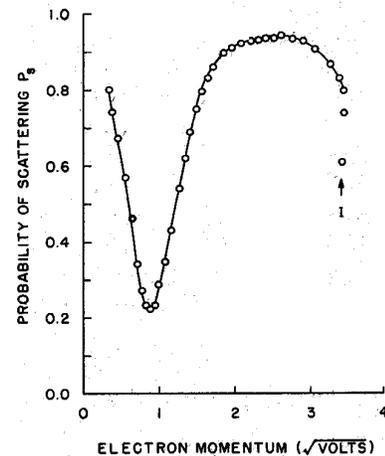


FIG. 4. The probability of scattering P_s as a function of $(V - V_s)^{1/2}$, where $V - V_s$ is the electron energy. Ionization occurs at "I".

the shield current and $f(V)$ is a geometrical factor which contains the ratio of the angle intercepted by the plate to the angle intercepted by the shield and a factor due to space charge effects near the cathode. To measure $f(V)$ we freeze out the xenon by dunking the top of the tube in liquid nitrogen. This reduces the xenon pressure to $\sim 10^{-3}$ Torr and P_s becomes very small so we get $f(V) \cong I_p^*/I_s^*$. Now we have $P_s = 1 - I_p I_s^*/I_s I_p^*$. Figure 3 shows that I_p has a maximum near 1 eV and that I_p/I_p^* approaches one there, indicating that there is very little scattering. At higher energies I_p/I_p^* is very small indicating a large probability of scattering. A plot of P_s calculated from the data using the above equation is shown in Fig. 4.

The probability of scattering is related to the mean free path by the relation $P_s = 1 - e^{-l/\lambda}$. For the 2D21 $l = 0.7$ cm so we can calculate λ . The cross section σ is related to λ by $n\sigma = 1/\lambda$, where n is the number of atoms per unit volume. A plot obtained from our values of P_s is shown in Fig. 5. A similar set of data for P_c ($P_c = P/\lambda$, where P is the pressure in Torr) given by Brode⁷ is shown in Fig. 6. In the 2D21 fairly large angular deflections must be produced to scatter an electron out of the beam (greater than ~ 0.2 rad) so the cross section measured in the 2D21 will be smaller than Brode's data.

II. EXPERIMENTAL DETAILS

The filament of the 2D21 is operated on 4 V dc. This is lower than the recommended value of

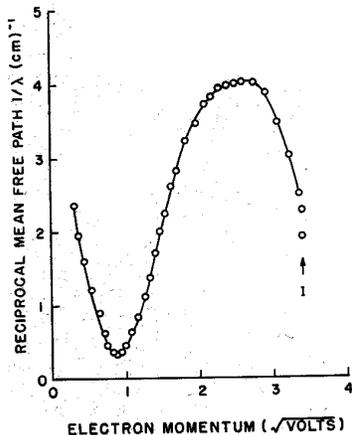


Fig. 5. The cross section times density $n\sigma = 1/\lambda$ as a function of $(V - V_s)^{1/2}$, where $V - V_s$ is the electron energy. Ionization occurs at "1".

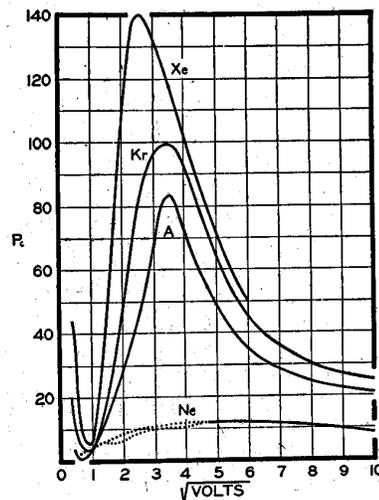


Fig. 6. The probability of collision P_c (= pressure times $n\sigma$) as a function of $(V)^{1/2}$ where V is the electron energy (from Brode see Ref. 2).

6.3 V, but tends to reduce space charge effects. Since the cathode temperature is lower, the thermal kinetic energy of the electrons is smaller and this will result in a narrower distribution of electron energies. The shield and plate currents are obtained by measuring the voltages V_s and V_p with two Keithley model 600A electrometers (see Fig. 2). The voltage source V is a well regulated and filtered supply which may be varied between 0-15 V. The electron energy plotted in the figures is $V - V_s$. We have not included a correction for the contact potential difference between the cathode and the shield. This contact potential difference is approximately 0.4 V and was measured by noting that ionization occurs when $V - V_s$ is 0.4 V less than the tabulated ionization potential. A similar value was obtained by measuring the value of V required to cut off the electron current to the shield. The voltages V_s and V_p range from a few millivolts to a few tenths of a volt. The data may be displayed on an oscilloscope by using an audio oscillator for the source V and for the x axis of the scope.

ACKNOWLEDGMENT

This experiment was suggested by Professor R. Weiss as a demonstration in an introductory course in quantum physics given by him at MIT.



2D21
THYRATRON

RELAY and GRID-CONTROLLED RECTIFIER SERVICE

Maximum Ratings, Absolute Values:

PEAK ANODE VOLTAGE:	650 max. volts
Forward:	1300 max. volts
Inverse:	-100 max. volts
GRID-No.2 (SHIELD-GRID) VOLTAGE:	-100 max. volts
Peak, before anode conduction:	-100 max. volts
Average, during anode conduction:	-10 max. volts
GRID-No.1 (CONTROL-GRID) VOLTAGE:	-100 max. volts
Peak, before anode conduction:	-10 max. volts
Average, during anode conduction:	-10 max. volts
CATHODE CURRENT:	0.5 max. amp
Peak:	0.1 max. amp
Average:	10 max. amp
Surge, for duration of 0.1 sec. max.:	+0.01 max. amp
Average:	+0.01 max. amp
GRID-No.1 CURRENT:	100 max. volts
Average:	25 max. volts
Surge, with respect to cathode:	-75 to +90 volts
HEATER POSITIVE VOLTAGE:	25 max. volts
HEATER NEGATIVE VOLTAGE:	-75 to +90 volts
AMBIENT TEMPERATURE RANGE:	-75 to +90 °C

Typical Operating Conditions for Relay Service:

RMS Anode Voltage:	117	400	volts
Grid-No.2 Voltage:	0	0	volts
RMS Grid-No.1 Bias Voltage:	5	5	volts
DC Grid-No.1 Bias Voltage:	-6	-6	volts
Peak Grid-No.1 Signal Voltage:	5	6	volts
Grid-No.1-Circuit Resistance:	1.0	1.0	megohms
Anode-Circuit Resistance:	1200	2000	ohms

Maximum Circuit Values:

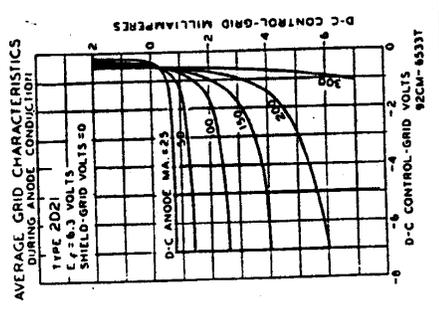
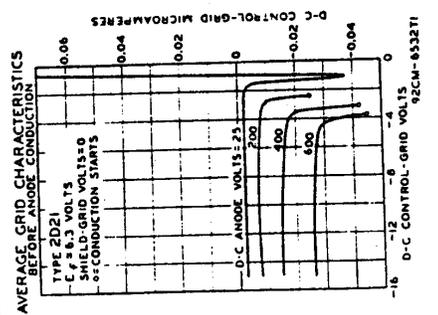
Grid-No.1-Circuit Resistance	10 max. megohms
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Ⓜ Averaged over any interval of 30 sec. max.
 Ⓞ Approximately 100 out of phase with the anode voltage.
 Ⓢ Sufficient resistance, including the tube load, must be used under any conditions of operation to prevent exceeding the current ratings.
 → Indicates a change.



2D21
THYRATRON

AVERAGE GRID CHARACTERISTICS



2D21
THYRATRON

GENERAL DATA

Electrical:			
Heater, for Unipotential Cathode:	Min.	Av.	Max.
Voltage (AC or DC):	5.7	6.3	6.9
Current, with heater volts = 6.3:	0.54	0.60	0.66
Cathode:			
Heating Time, prior to tube conduction:	10		sec
Direct Interleaved Capacitance (Approx.):			
Grid No.1 to Anode:	0.026		μf
Input:	2.4		μf
Output:	1.6		μf
Ionization Time (Approx.):			
For conditions: dc anode volts 100; grid-No.1 square-pulse volts = 90; peak anode amp. during conduction 0.5:	0.5		μsec
Deionization Time (Approx.):			
For conditions: dc anode volts = 125; grid-No.1 volts = -100; grid-No.1 resistor (ohms) = 1000; dc anode amp. = 0.1:	35		μsec
For conditions: dc anode volts = 125; grid-No.1 volts = -10; grid-No.1 resistor (ohms) = 1000; dc anode amp. = 0.1:	75		μsec
Maximum Critical Grid Current, with ac anode supply volts (rms) = 450, and average anode amp. = 0.1:	0.5		μamp
Anode Voltage Drop (Approx.):	8		volts
Grid-No.1 Control Ratio (Approx.) with grid-No.1 resistor (megohms) = 0; grid-No.2 volts = 0:	250		
Grid-No.2 Control Ratio (Approx.) with grid-No.1 resistor (megohms) = 0; grid-No.2 resistor (megohms) = 0; grid-No.1 volts = 0:	1000		
Ⓞ Without external shield.			

Mechanical:

- Mounting Position: Any
- Maximum Overall Length: 2-1/8"
- Maximum Sealed Length: 1-7/8"
- Length, Base Seat to Bulb Top (excluding tip): 1-1/2" ± 3/32"
- Bulb Maximum Diameter: 3/4"
- Base: T-5-1/2"
- Basing Designation for BOTTOM VIEW: Snail-Button Miniature 7-Pin
- Pin 1 - Grid No. 1
- Pin 2 - Cathode
- Pin 3 - Heater
- Pin 4 - Heater
- Pin 5 - Grid No. 2
- Pin 6 - Anode
- Pin 7 - Grid No. 2

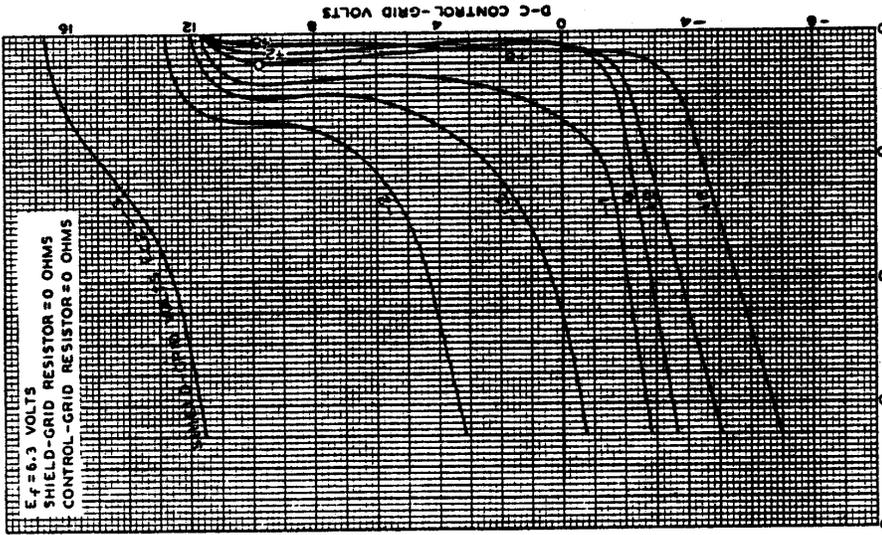




2D21

THYRATRON

AVERAGE CONTROL CHARACTERISTICS



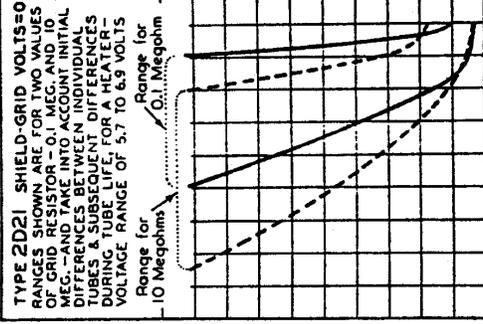
MAY 2, 1944
 RCA VACUUM DIVISION
 92CM-6531R1
 BASED ON CHARACTERISTICS OF TYPICAL TUBES, SEE NOTE



2D21

THYRATRON

OPERATIONAL RANGE OF CRITICAL GRID VOLTAGE



Range for 10 Megohms
 Range for 0.1 Megohm
 92CM-6534T2

JUNE 15, 1948
 TUBE DEPARTMENT
 BASED ON CHARACTERISTICS OF TYPICAL TUBES, SEE NOTE
 CE-6534T2

Procedure

1. Read the article by S.G. Kukolich in the Am. Jour. Phys. 36, 1968 and understand the one-dimensional scattering from a square well.
2. Set up the circuit as in the diagram on page 4.
3. Allow 5 minutes for the tube filament, cathode and multimeters to heat up and become stable.
4. Measure the voltages V_s and V_p as a function of the cathode to shield voltage ($V - V_s$) with the thyratron at room temperature. Try using values of ($V - V_s$) as follows:

from	0.25	to	1.00	volts in steps of	0.05	volts
	1.00	to	2.00	volts in steps of	0.1	volts
	2.00	to	3.00	volts in steps of	0.2	volts
	3.00	to	5.00	volts in steps of	0.5	volts
	5.00	to	13.00	volts in steps of	1.0	volts

The purpose of the uneven steps is to give the best detail between 0.3 and 1.0 on the plot of $\sqrt{V - V_s}$. You will find that you cannot increase ($V - V_s$) to 13V because the Xenon gas begins to ionize. Do not increase V_s above 3V. Estimate the voltage at which ionization occurs and compare with the accepted value of 12.13 Volts. The difference is due to the contact potential difference between cathode and shield.

5. Turn off the filament and gently immerse only the lower blackened part of the thyratron in liquid nitrogen. Allow it to cool for 15 minutes then turn on the filament again and allow a further 5 minutes for temperatures to stabilize. The Xenon will have condensed and frozen at the cold end of the tube.
6. Repeat measurements of Step 4 above at the same values of ($V - V_s$) to obtain V_s^* and V_p^* . Adjust the tube from time to time to keep the lower end in the liquid nitrogen.
7. Plot I_p and I_p^* against $\sqrt{V - V_s}$.
8. Calculate the probability of transmission (no scattering):

$$T = \frac{I_p I_s^*}{I_s I_p^*}.$$

Since $V_p = I_p R_p$

$$V_p^* = I_p^* R_p$$

$$V_s = I_s R_s$$

$$V_s^* = I_s^* R_s,$$

it is easier to calculate:

$$T = \frac{V_p V_s^*}{V_s V_p^*}.$$

Plot T against $\sqrt{V - V_s}$ (which is proportional to the electron momentum).

Plot T against $V - V_s$ (which is proportional to the electron energy).

Note the value of $(V - V_s)$ corresponding to maximum T . Correct your result for the contact potential difference. The contact potential is best determined by measuring the value of $V - V_s$ which makes the current I_p equal to zero. If there were no contact potential, $I_p = 0$ would correspond to $V - V_s = 0$. You will find that the required value of $V - V_s$ to make $I_p = 0$ is a reverse polarity. The value of this offset voltage is the contact potential.

9. Assume that the diameter of a Xenon atom is about 2.8 Å (Xenon is smaller than Cesium (5.5 Å) because Xenon has closed shells). From your data and using one-dimensional Quantum Mechanics estimate the average depth of the square well seen by the electrons.
10. A somewhat more realistic result for the depth of the square well seen by the electrons can be made by using the three-dimensional square well as a model. Theory predicts that the scattering will be a minimum when the phase shift δ_0 of the $\ell = 0$ partial wave is $n\pi$ provided that all other partial wave contributions are negligible. The condition that the wave function and its derivative must be continuous at the boundary $r = a$ then becomes

$$k_2 a \tan k_1 a = k_1 a \tan k_2 a$$

where $k = \frac{2\pi}{\lambda}$, $\lambda_1 =$ wave length of the electron inside the square well, and $\lambda_2 =$ wave length of the free electron. Use this relation to make another estimate of the depth of the square well.

References

- [1] “Demonstration of the Ramsauer - Townsend Effect in a Xenon Thyatron”, S.G. Kukolich, Am. J. Phys. **36**, 1968, pages 701 - 701, included in this description.
- [2] *An Introduction to Quantum Physics* (Norton, 1978), A.P. French and E.F. Taylor,
- [3] *Quantum Mechanics, 3rd Ed.* (Wiley, 1998), E. Merzbacher.
- [4] *Modern Physics and Quantum Mechanics* (Saunders, 1971), E.E. Anderson.

Appendix A (From French and Taylor, Section 9-4, pp 379-383)

9-4 SCATTERING BY A ONE-DIMENSIONAL WELL

As a second example of scattering, let us consider the situation shown in Figure 9-5a. Particles of total energy E (relative to a zero-potential level represented by region I) encounter a potential “hole” of depth V_0 and width L . Partial reflection and transmission must be assumed to take place at both sides of the well. In regions I and III the wave numbers have the same value, $k_1 = \sqrt{2mE} / \hbar$; in region II there is a

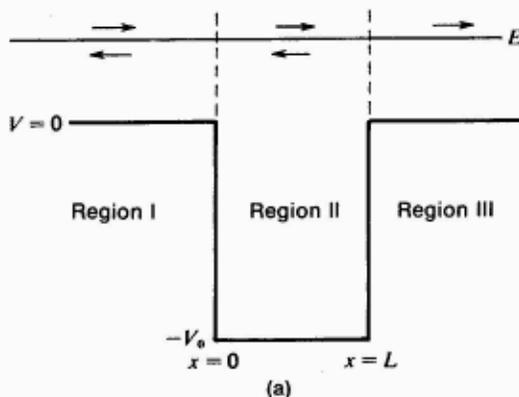
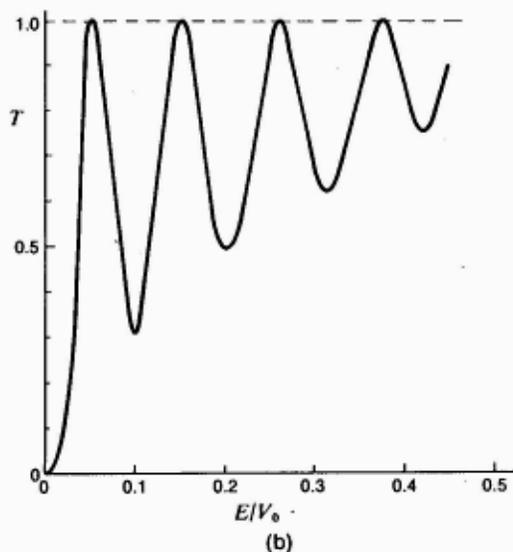


Fig. 9-5

(a) Steady-state situation for one-dimensional scattering of particles of unique energy E by a square well.
 (b) Transmission coefficient T of square well as a function of incident particle energy, calculated for the dimensionless parameter $L\sqrt{2mV_0}/\hbar$ equal to 20.5π . Note resonances giving 100 percent transmission at certain energies.



larger wave number, $k_2 = \sqrt{2m(E + V_0)} / \hbar$

Appropriately extending the analysis of Section 9-2, we can write

$$\begin{aligned}
 \Psi_I(x) &= A_0 e^{ik_1 x} + A e^{-ik_1 x} \\
 \Psi_{II}(x) &= B e^{ik_2 x} + C e^{-ik_2 x} \\
 \Psi_{III}(x) &= D e^{ik_1 x}
 \end{aligned}
 \tag{9-10}$$

You can easily verify that the result of applying the continuity conditions on Ψ and $d\Psi/dx$ at $x=0$ and $x=L$ is the following set of equations:

$$\begin{aligned}
 A_0 + A &= B + C \\
 ik_1 A_0 - ik_1 A &= ik_2 B - ik_2 C \\
 B e^{ik_2 L} + C e^{-ik_2 L} &= D e^{ik_1 L} \\
 ik_2 B e^{ik_2 L} - ik_2 C e^{-ik_2 L} &= ik_1 D e^{ik_1 L}
 \end{aligned}
 \tag{9-11}$$

Here we have four equations relating five undetermined coefficients; this is enough information to obtain the values of A, B, C, and D as fractions of A_0 . To calculate the transmission coefficient T of the well we need to find the value of D/A_0 . The algebra of this is not difficult. From the first pair of Eqs. 9-11 we easily find:

$$2k_1 A_0 = (k_2 + k_1)B - (k_2 - k_1)C$$

From the second pair of Eqs. 9-11 we can find B and C in terms of D:

$$\begin{aligned}
 B &= \frac{k_2 + k_1}{2k_2} D e^{ik_1 L} e^{-ik_2 L} \\
 C &= \frac{k_2 - k_1}{2k_2} D e^{ik_1 L} e^{ik_2 L}
 \end{aligned}$$

Substituting these expressions for B and C in the preceding equation then leads to the result:

$$4k_1 k_2 A_0 = [(k_2 + k_1)^2 e^{-ik_2 L} - (k_2 - k_1)^2 e^{ik_2 L}] D e^{ik_1 L} \tag{9-12}$$

The quantity $|D/A_0|^2$ is the ratio of probability density in the transmitted beam to that in the beam incident on the well. Since, however, the potential energy is the same on both sides of the well (and hence k has the same value) the ratio $|D/A_0|^2$ as given by Eq. 9-12 is also the ratio of transmitted current to incident current. That is, the transmission coefficient T is equal to $|D/A_0|^2$. The form of its

variation with particle energy is shown in Figure 9-5b.

Without evaluating the general result (done in exercise 9-11), we can identify certain properties of this scattering system:

1. For $k_1 \ll k_2$ (incident particle energy E much less than well depth V_0) we have:

$$4k_1k_2A_0 \approx k_2^2(e^{-ik_2L} - e^{ik_2L})De^{ik_1L} = -(2ik_2^2 \sin k_2L)De^{ik_1L}.$$

Therefore:

$$T \approx \frac{4k_1^2}{k_2^2 \sin^2 k_2L}.$$

Here, k_1 is proportional to \sqrt{E} , and $k_2 (= \sqrt{2m(V_0 + E)} / \hbar)$ is approximately constant as E is varied. Hence $T \sim E$ (the transmission of the well rises linearly with incident particle energy.³)

2. For $E \gg V_0$, we have $k_2 \approx k_1$, in which case:

$$4k_1^2A \approx [(2k_1)^2 e^{-ik_1L}]De^{ik_1L} = 4k_1^2D.$$

Therefore $T \approx 1$.

Thus for incident particle energies much bigger than the well depth, the transmission approaches 100 percent.

3. For $k_2L = n\pi$, we have a very interesting resonance condition. For values of k_2 satisfying this condition (n integral) we have:

$$\begin{aligned} e^{ik_2L} = e^{-ik_2L} &= +1 && \text{(n even)} \\ e^{ik_2L} = e^{-ik_2L} &= -1 && \text{(n odd)} \end{aligned}$$

³This result does not hold if $k_2L \rightarrow n\pi$ as $E \rightarrow 0$. In that case $T \rightarrow 1$ as $E \rightarrow 0$, in the manner described in property 3 below.

Under these conditions, Eq. 9-8 gives us (exactly):

$$4k_1k_2A_0 = \pm[(k_2 + k_1)^2 - (k_2 - k_1)^2]De^{ik_1L}.$$

Therefore:

$$A_0 = \pm De^{ik_1L} \text{ and } T = 1.$$

Thus for all energies such that $k_2L = n\pi$ the well is completely transparent to the incident particles. The condition for this to happen ($k_2 = n\pi / L$) corresponds to the width L of the well being equal to an integral number of half-wavelengths λ_2 of the wave function Ψ_{II} inside the well. We have $k_2 = 2\pi / \lambda_2$, and hence:

$$2L = n\lambda_2.$$

This behavior is closely analogous to the selective transmission of light of particular wavelengths by a thin layer of glass or dielectric—an effect that is exploited in optical interference filters, which by a careful choice of thickness transmit light within a narrow band of wavelengths with far less attenuation than occurs with normal colored filters (which work by selective absorption).

The wave-mechanical transparency of a potential well is observed in the scattering of electrons by noble-gas atoms, and is known as the Ramsauer effect. It manifests itself as a minimum in the cross section (target area) presented by atoms to incident electrons at a certain value of the electron energy. If an atom of radius R could be regarded as a simple rectangular well of width $L = 2R$, the above analysis would imply a minimum in the cross section for $\lambda_2 = 4R$, corresponding to an electron kinetic energy inside the well equal to $\frac{h^2}{2m\lambda_2^2}$, or $\frac{h^2}{32mR^2}$. For

$R \approx 1 \text{ \AA}$, this would give a value of about 10 eV. Actual experiments (Figure 9-6) show a minimum cross section for an incident electron energy of only about 1 eV. A full 3 dimensional model is required for a more accurate result.

Fig. 9-6 Ram-
sauer effect. Mini-
mum in cross sec-
tion for scattering of
electrons by xenon
at about 1 eV.
[After S. G. Kuko-
lich, *Am. J. Phys.*
36, 701 (1968).]

